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Grade/Class : 11/.........   **Mathematics Teacher** : ....

MATHEMATICS
FINAL ASSESSMENT
Paper 2
ANSWER BOOKLET

150

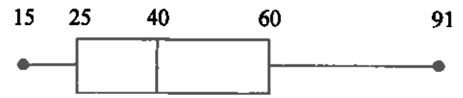
QUESTION 1

1.1.

1	2	3	4	5	6	7	8	9	10
10,5	15	17,7	20	21	22	23,6	24	26,7	29

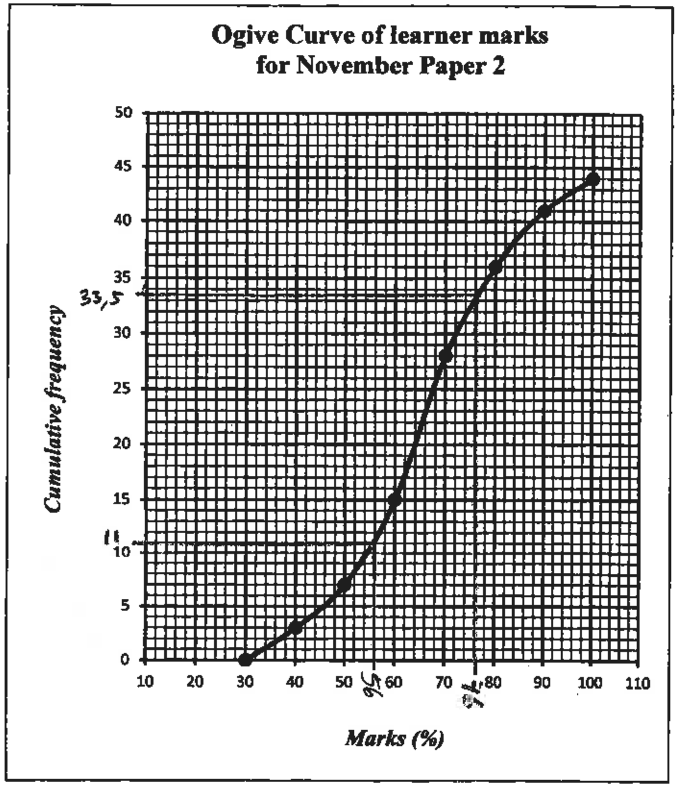
1.1.1	(a) $\bar{x} = 20,95$	1
	(b) $M = T_{\frac{1}{2}(1+10)}$	
	$= T_{5,5}$	
	$= \frac{21+22}{2}$	
	$= 21,5$	1
	(c) $\sigma = 5,21$	1
	(d) Variance $= 5,21^2$	
	$= 27,14$	1
1.1.2.	$\bar{x} - 1\sigma = 20,95 - 5,21$	
	$= 15,74$	
	$\therefore$ 10,5 15	
	$\therefore$ 2 values	2
1.1.3.	$\bar{x} - M = 20,95 - 21,5$	
	$= -0,55$	
	< 0	
	$\therefore$ data is negatively skewed	
	(skewed to the left).	2
	$M - Q_1$ $Q_3 - M$	
	$= 21,5 - 17,7$ $= 24 - 21,5$	
	$= 3,8$ $= 2,5$	
	$3,8 > 2,5$	
	$\therefore$ left	

1.2.



1.2.1.	$IQR = 60 - 25$	
	$= 35$ ✓	
	$UF = Q_3 + 1{,}5 \cdot IQR$	
	$= 60 + 1{,}5 \cdot 35$	
	$= 112{,}5$ ✓	
	$91 < 112{,}5$	
	∴ 91 is NOT ✓ an outlier	3
1.2.2.	40 % − 60 % M − Q_3	
	∴ ✓ 25 % of learners	
	$= \frac{25}{100} \times 28$	
	= ✓ 7 learners	2

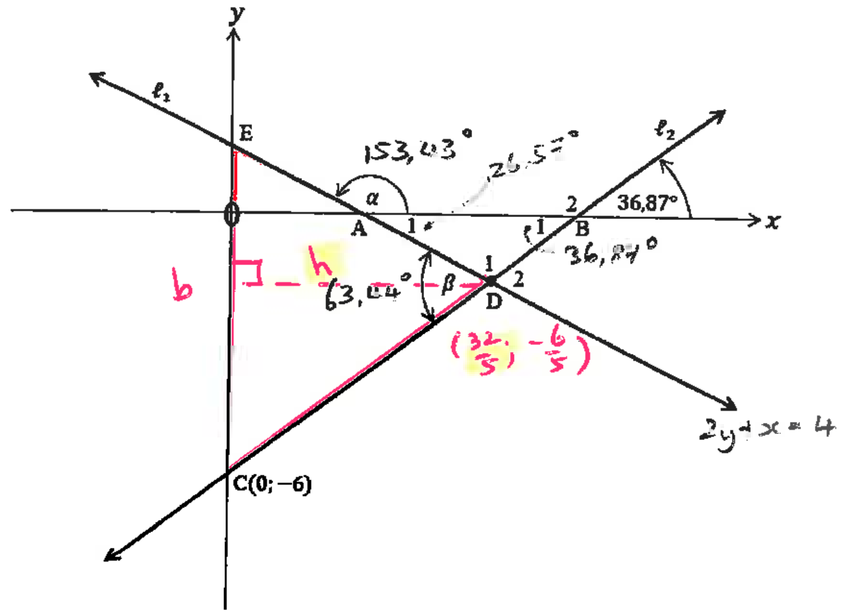
QUESTION 2



2.1.	44 Maths learners	1
2.2.1.	$T_1; \ldots; T_{44}$ $\quad M = T_{\frac{1}{2}(1+44)}$	
	$= T_{22,5}$	
	$T_{23}; \ldots; T_{44}$ $\quad Q_3 = T_{\frac{1}{2}(23+44)}$	
	$= T_{33,5}$	
	$\approx 76\ \%$	2

2.2.2.	$\leq 56\ \% = 11$	
	$\therefore > 56 = 44 - 11$	
	$= 33$ learners	2
2.3.	Modal class is $60 < x \leq 70$	1
2.4.	$T_1; \ldots; T_{44}$	
	$P_{20} = T_{\frac{20}{100}(1+44)}$	
	$= T_9$	
	$\therefore$ 9th position	1

QUESTION 3



3.1.1	$2y + x = 4$	
	$2y = -x + 4$	
	$\div 2: \quad y = -\frac{1}{2}x + 2$	
	$m = \tan\alpha$	
	$-\frac{1}{2} = \tan\alpha$ ✓	
	ref $\angle = 26,56...°$	
	tan − in	
	II : $\alpha = 153,43°$ ✓	2

3.1.2.	$\hat{B}_1 = 36,87°$ ✓ vert opp ∠s =	
	$\hat{A}_1 = 26,57°$ ✓ ∠s on str line = 180°	
	$\therefore \beta = 63,44°$ ✓ ext ∠ Δ	3
3.2.1.	$m = \tan\theta$	
	$= \tan 36,87°$	
	$= 0,75$ ✓	1
3.2.2.	$y = 0,75x - 6$ ✓	1
3.2.3.	D $\ell_1 \cap \ell_2$	
	$y = -\frac{1}{2}x + 2 \qquad y = 0,75x - 6$	
	$-\frac{1}{2}x + 2 = 0,75x - 6$ ✓	
	$-\frac{5}{4}x = -8$	
	$x = \frac{32}{5}$	
	$y = -\frac{1}{2}(\frac{32}{5}) + 2$	
	$= -\frac{6}{5}$	
	$\therefore D(\frac{32}{5}; -\frac{6}{5})$ ✓ ✓	3
	6,4 −1,2	

3.3. $y_C = -6$ $\qquad y_E = 2$

$\therefore CE = 2 - (-6)$

$= 8$

$x_D = \frac{32}{5}$

$\therefore h = \frac{32}{5}$

$\therefore$ area $\triangle CDE$

$= \frac{1}{2}(8)\left(\frac{32}{5}\right)$

$= \frac{128}{5}$ units^2 25,6

(3)

OR

$E(0;2) \qquad D\left(\frac{32}{5}; -\frac{6}{5}\right)$

$DE = \sqrt{\left(-\frac{6}{5} - 2\right)^2 + \left(\frac{32}{5} - 0\right)^2}$

$= \sqrt{\frac{256}{5}}$ 7,155...

$C(0;-6) \qquad D\left(\frac{32}{5}; -\frac{6}{5}\right)$

$CD = \sqrt{\left(-\frac{6}{5} - (-6)\right)^2 + \left(\frac{32}{5} - 0\right)^2}$

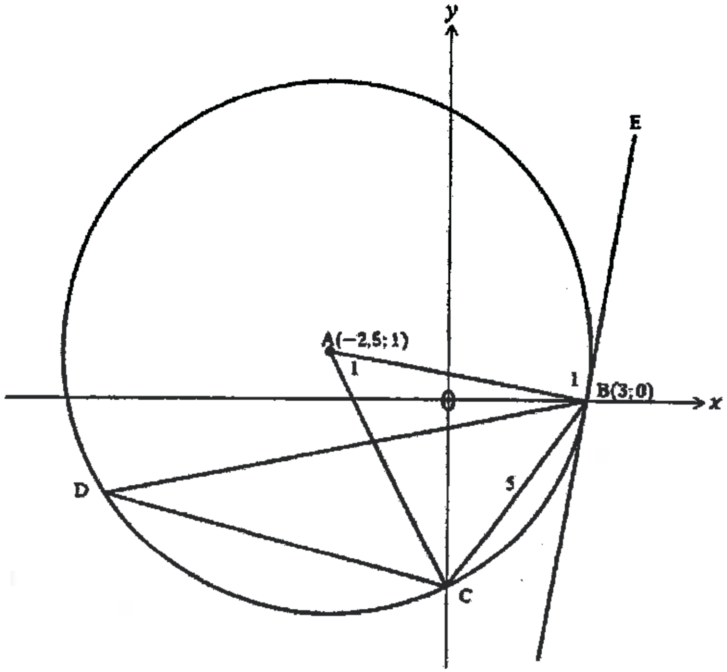
$= 8$

area $= \frac{1}{2} \cdot \sqrt{\frac{256}{5}} \cdot 8 \cdot \sin 63{,}44°$

$= 25{,}6\ u^2$

ADDITIONAL SPACE

QUESTION 4



4.1.1.	m_{AB} $\qquad A(-2,5\,;1)\quad B(3;0)$	
	$= \dfrac{0-1}{3-(-2,5)}$	
	$= -\dfrac{2}{11}$ $\qquad -0,18$	(2)
4.1.2.	$\hat{B}_1 = 90^\circ$ $\qquad$ tan ⊥ rad	(1)

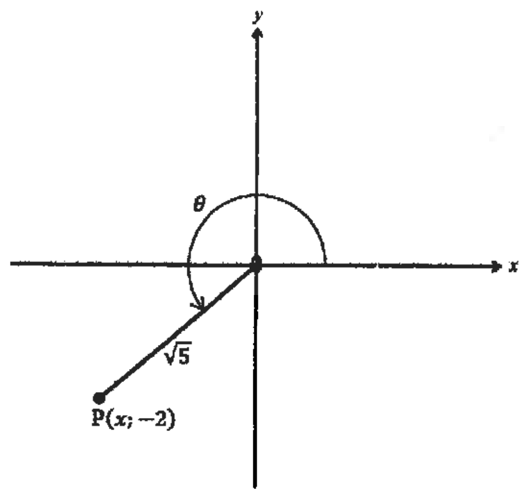
4.1.3.	$m_{tan} = \frac{11}{2}$ $\qquad$ tan ⊥ rad	
	$y = \frac{11}{2}x + c$	
	sub $B(3;0)$	
	$0 = \frac{11}{2}(3) + c$	
	$-\frac{33}{2} = c$ $\qquad -16,5$	
	$\therefore\ y = \frac{11}{2}x - \frac{33}{2}$	(3)
4.2.1.	AB $\qquad A(-2,5\,;1)\quad B(3;0)$	
	$= \sqrt{(0-1)^2 + (3-(-2,5))^2}$	
	$= \sqrt{\frac{125}{4}}$ $\qquad \frac{5\sqrt{5}}{2}$	(2)
	$\sqrt{31,25}$	
4.2.2.	$AB = AC$ $\qquad$ radii	
	$= \sqrt{\frac{125}{4}}$	
	$BC^2 = AC^2 + AB^2 - 2\,AC.AB.\cos\hat{A}_1$	
	$5^2 = \left(\sqrt{\frac{125}{4}}\right)^2 + \left(\sqrt{\frac{125}{4}}\right)^2 - 2\sqrt{\frac{125}{4}}\sqrt{\frac{125}{4}}\cos\hat{A}_1$	
	$-\frac{75}{2} = -\frac{125}{2}\cos\hat{A}_1$	
	$\therefore\ \cos\hat{A}_1 = \frac{3}{5}$	
	$\hat{A}_1 = 53,13...^\circ$	
	Cos + m	
	$\therefore\ \hat{A}_1 = 53,13^\circ$	(4)

4.3.	$\hat{D} = 26{,}57°$ ∠ @ centre = 2∠ @ circumf	2
4.4.	$F(x;y)$ $A(-2{,}5;1)$ $B(3;0)$	
	$BA = AF$ radii	
	$\frac{x_F+3}{2} = -2{,}5$ $\frac{y_F+0}{2} = 1$	
	$x_F + 3 = -5$ $y_F = 2$	
	$x_F = -8$	
	$\therefore F(-8;2)$	2

	ADDITIONAL SPACE	

QUESTION 5

5.1.



5.1.1.	$\sin\theta = \frac{-2}{\sqrt{5}}$ ✓ $\quad \frac{y}{r}$	1
5.1.2.	$\tan(1980° - \theta)$	
	$= \tan(180° - \theta)$ ✓ $\quad -5.360°$	
	$= -\tan\theta$ ✓	
	$= -\frac{y}{x}$	
	$x^2 + (-2)^2 = (\sqrt{5})^2$ Pyth	
	$x^2 + 4 = 5$	
	$x^2 = 1$	
	$x = \pm 1$	
	$\therefore x = -1$ ✓ reject +	

	$\therefore = \frac{-2}{-1}$ ✓	
	$= -2$ ✓	5
5.2.	• $\cos(180° + \theta) = -\cos\theta$	
	• $\sin(270° + \theta) = -\cos\theta$	
	• $\cos(-\theta) = \cos\theta$	
	$\frac{2\cos^2(180°+\theta) - 5\sin(270°+\theta) - 3}{3\cos^2\theta + \cos(-\theta) + 3\sin^2\theta}$	
	$= \frac{2(-\cos\theta)^2 - 5(-\cos\theta) - 3}{\cos\theta + 3(\cos^2\theta + \sin^2\theta)}$ ✓ ✓ ✓ ✓ cf	
	$= \frac{2\cos^2\theta + 5\cos\theta - 3}{\cos\theta + 3.1}$	
	$= \frac{(2\cos\theta - 1)(\cos\theta + 3)}{\cos\theta + 3}$ ✓ factors ✓	
	$= 2\cos\theta - 1$ ✓ ans	7
5.3.	LHS $\qquad$ RHS	
	$= \frac{1}{\tan\theta} \qquad = \frac{\sin^2\theta}{\tan\theta - \sin\theta\cos\theta}$	
	$= \frac{1}{\frac{\sin\theta}{\cos\theta}}$ ✓ $\qquad = \frac{\sin^2\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta\cos\theta}$	

$= 1 \times \frac{\cos\theta}{\sin\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta - \frac{\sin\theta.\cos^2\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{1} \times \frac{\cos\theta}{\sin\theta(1-\cos^2\theta)}$

$= \frac{\sin^2\theta}{1} \times \frac{\cos\theta}{\sin\theta.\sin^2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$\therefore$ LHS = RHS

(5)

5.4.1 $3\cos\theta + 2\sin 170° = 0$

$\cos\theta = -0{,}115\ldots$

ref $\angle = 83{,}35\ldots°$

Cos – ive $(k \in \mathbb{Z})$ NB

II : $\theta = 96{,}65° + k.360°$

III : $\theta = 263{,}35° + k.360°$

(3)

5.4.2 $\sin(2\theta + 10°) + \cos(\theta - 50°) = 0$

Let $A = 2\theta + 10°$

$B = \theta - 50°$

$\sin A + \cos B = 0$

$\therefore \sin A = -\cos B$

$\sin(270° - B)$ III $\sin(270° + B)$ IV

$(k \in \mathbb{Z})$ NB

$\sin A = \sin(270° - B)$ or $\sin A = \sin(270° + B)$

• $A = 270° - B + k360°$; $A = 270° + B + k360°$

• $2\theta + 10° = 270° - (\theta - 50°) + k\,360°$

$2\theta + 10° = 270° - \theta + 50° + k\,360°$

$3\theta = 310° + k.360°$

$\theta = 103{,}33° + k\,120°$

: $2\theta + 10° = 270° + (\theta - 50°) + k.360°$

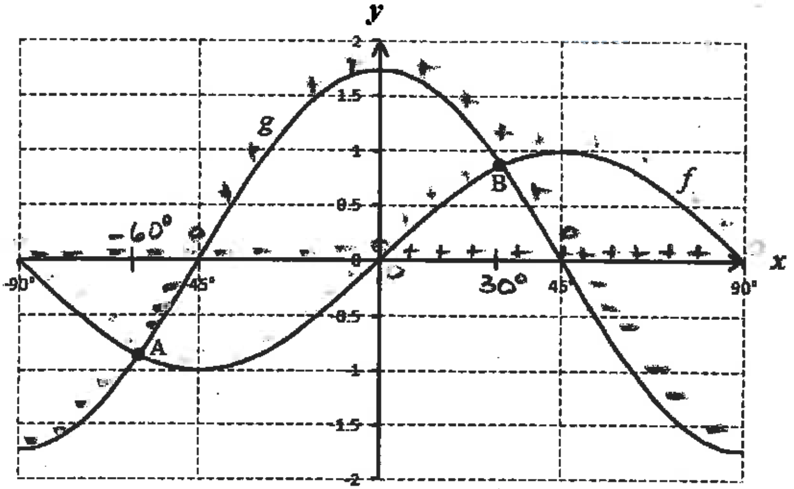
$2\theta + 10° = 270° + \theta - 50° + k.360°$

$\theta = 210° + k.360°$

(5)

NB: $k \in \mathbb{Z}$ must be stated in each answer

QUESTION 6



6.1	$\sin 2x = \sqrt{3}.\cos 2x$	
	let $A = 2x$	
	$\sin A = \sqrt{3}\ \cos A$	
	$\div \cos A$: $\frac{\sin A}{\cos A} = \frac{\sqrt{3}\cos A}{\cos A}$	
	$\tan A = \sqrt{3}$ ✓	
	ref ^ $= 60°$	
	tan + in' $(k \in \mathbb{Z})$	
	I : $A = 60° + k.180°$ ✓	
	$2x = 60° + k.180°$	
	$x = 30° + k.90°$ ✓	3

6.2.1.	$g(x) < 0$	
	$y_g < 0$	
	y_g $-$ ✓A ✓A	
	$\therefore x \in [-90°; -45°)$ or $(45°; 90°]$	2
6.2.2.	$f(x).g(x) \geq 0$	
	$y_f . y_g \geq 0$	
	$y_f . y_g$ $\overset{+}{0}$	
	$\therefore x \in [-90°; -45°]$ or $[0°; 45°]$ or $x = 90°$	3
	✓A ✓A ✓A	
6.2.3.	$f(x) - g(x) > 0$	
	$y_f - y_g > 0$	
	$y_f > y_g$	
	f above g	
	$\therefore x \in [-90°; -60°)$ or $(30°; 90°]$	2
	✓A ✓A	
6.2.4.	$x . f(x) \leq 0$	
	$x . y_f \leq 0$	
	$x . y_f$ $\overline{0}$	
	$\therefore x = -90°$ or $x = 0°$ or $x = 90°$	3
	✓ ✓ ✓	

6.3. f $y = \sin 2x$ $y \in [-1; 1]$

$y = 5 \sin 2x$ $y \in [-5; 5]$

h $y = 5 \sin 2x - 1$ $y \in [-6; 4]$

CV

6.4. f : $y = \sin 2x$

i : $y = -\sin(2x - 50°)$

$= -\sin 2(x - 25°)$

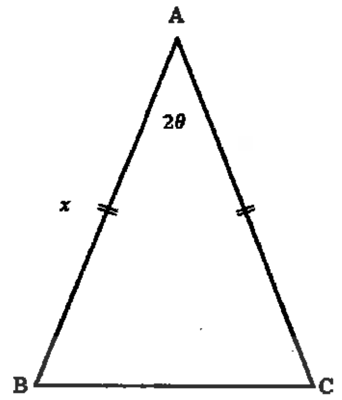
∴ f

- translated 25° horizontally to the right
- reflected in the x-axis

to become i.

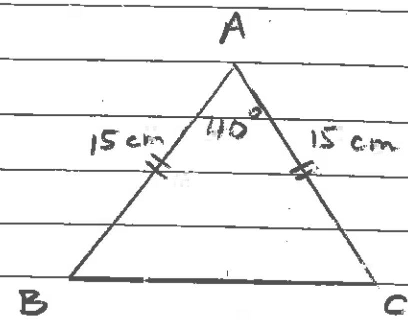
ADDITIONAL SPACE

QUESTION 7



7.1.1.	$\hat{B} = \hat{C}$ ∠s opp = sides	
	$2\hat{C} + 2\theta = 180°$ sum ∠s in Δ = 180°	
	÷ 2 :	
	$\hat{C} + \theta = 90°$	
	$\hat{C} = 90° - \theta$	2
7.1.2.	$\frac{BC}{\sin 2\theta} = \frac{x}{\sin(90° - \theta)}$	
	$\frac{BC}{\sin 2\theta} = \frac{x}{\cos\theta}$	
	$BC = \frac{x \sin 2\theta}{\cos\theta}$	3

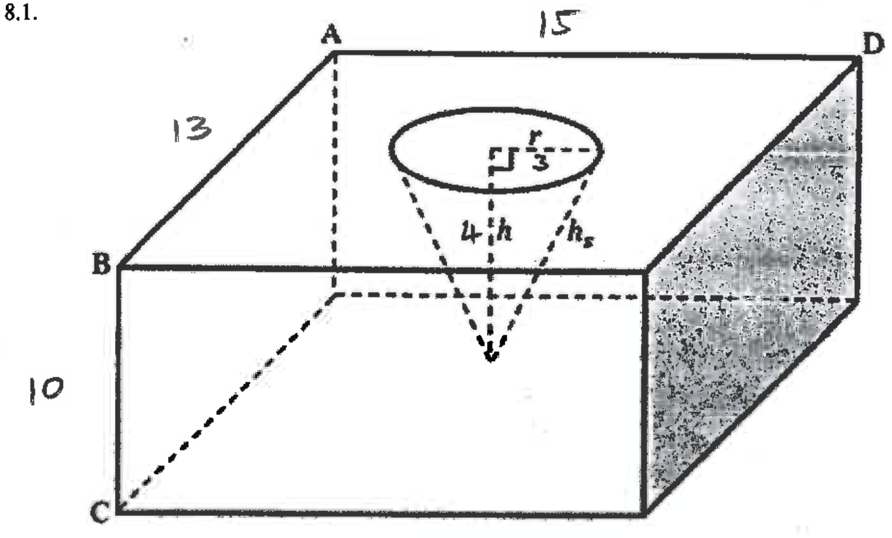
OR

7.2.		
	area Δ ABC	
	$= \frac{1}{2}(15)(15)\sin 40°$	
	$= 72,31$ units²	2
	$BC^2 = x^2 + x^2 - 2x \cdot x \cos 2\theta$	
	$= 2x^2 - 2x^2 \cos 2\theta$	
	$= 2x^2(1 - \cos 2\theta)$	
	$BC = \sqrt{2x^2(1 - \cos 2\theta)}$	
	$= \sqrt{x^2}\sqrt{2(1 - \cos 2\theta)}$	
	$= x\sqrt{2(1 - \cos 2\theta)}$	3

QUESTION 8

8.1.

cm



8.1.1 $V = V_{rrp} - V_c$

$= (15 . 13) \times 10 - \frac{1}{3}(\pi (3)^2) \times 4$

$= 1950 - 12\pi$ 37,699...

$= 1\,912,30 \text{ cm}^3$ 3

8.1.2. $4^2 + 3^2 = h_s^2$ Py th

$25 = h_s^2$

$5 = h_s$ 5   right -

TSA

$= 2 \times \boxed{}^{10}_{13} + 2 \times \boxed{}^{10}_{15} + \boxed{}^{13}_{15} + \boxed{}^{13}_{15} + \pi r h_s$

all 3

$= 2\times(13\times10) + 2\times(15\times10) + 13.15 + (13\times15 - \pi(3)^2) + \pi.3.5$

$= 260 + 300 + 195 + (195 - 9\pi) + 15\pi$

$= 755 + 166,725... + 47,123...$

$= 968,85 \text{ cm}^2$ 5

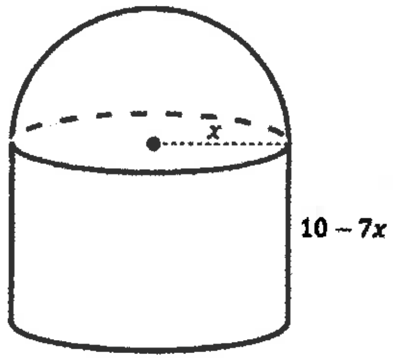
OR

$2(13.10) + 2(15.10) + 2(13.15) - \pi.3^2 + \pi.3.5$

$= 950 - 28,27... + 47,12...$

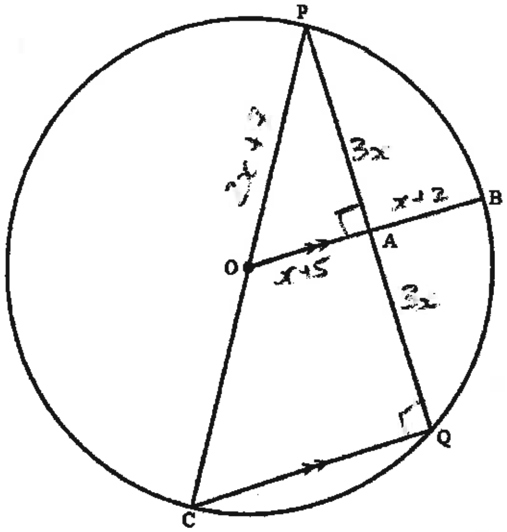
$= 968,85 \text{ cm}^2$

8.2.



8.2.1.	$TSA = \frac{1}{2}(4\pi x^2) + \pi x^2 + 2\pi x(10-7x)$	
	$= 2\pi x^2 + \pi x^2 + 20\pi x - 14\pi x^2$	
	$= \underline{20\pi x - 11\pi x^2 \quad unit^2}$	3
8.2.2.	$TSA = -11\pi x^2 + 20\pi x$	
	TSA 	
	x	
	$x = \frac{-(20\pi)}{2(-11\pi)}$	
	$= \frac{10}{11}$	
	sub	
	$TSA_{max} = -11\pi\left(\frac{10}{11}\right)^2 + 20\pi\left(\frac{10}{11}\right)$	
	$= \underline{28{,}56 \quad units^2}$	3

QUESTION 9

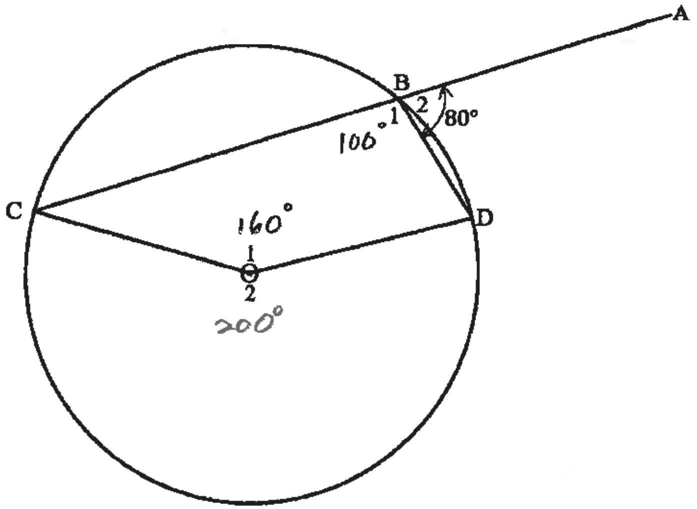


9.1.	$\hat{Q} = 90°$ ✓S ✓R ∠ in semi ⊙ = 90°		
	$O\hat{A}P = 90°$ ✓R corr ∠s =, OA ∥ CQ		(3)
9.2.	PA = AQ	line from centre ⊙	
	= 3x ✓R	⊥ to chord	
	OA = 2x+7 − (x+2)	radii	
	= 2x + 7 − x − 2		
	= x + 5 ✓S		

	$(2x+7)^2 = (3x)^2 + (x+5)^2$ ✓SR Pythag	
	$4x^2 + 28x + 49 = 9x^2 + (x^2 + 10x + 25)$	
	$4x^2 + 28x + 49 = 9x^2 + x^2 + 10x + 25$	
	$0 = 6x^2 - 18x - 24$	
	$\div 6: \quad 0 = x^2 - 3x - 4$ ✓std	
	$0 = (x-4)(x+1)$ ✓fac	
	$\therefore x = 4$ or $\cancel{-1}$	
	✓ reject	
	ans with selection	(6)

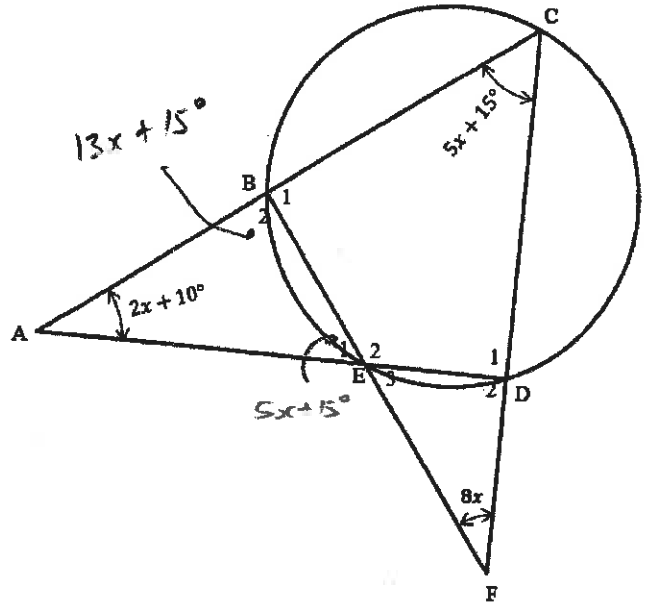
QUESTION 10

10.1



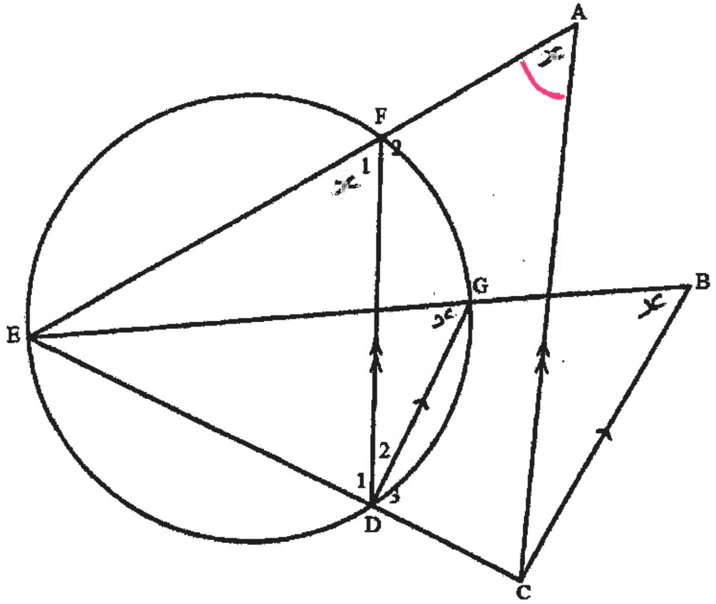
	$\hat{B}_1 = 100°$ ✓ ∠'s on str line = 180°	
	$\hat{O}_2 = 200°$ ✓✓ ∠ @ centre = 2x ∠ @ circum	
	$\hat{O}_1 = 160°$ ✓ ∠'s in a rev = 360°	4

10.2



	$\hat{E}_1 = 5x + 15°$ ✓✓ ext ∠ cyclic quad	
	$\hat{B}_2 = 5x + 15° + 8x$ ext ∠ Δ	
	$= 13x + 15°$ ✓	
	$2x + 10° + 13x + 15° + 5x + 15° = 180°$ ✓	
	sum ∠'s in Δ = 180°	
	$20x = 140°$	
	$x = 7°$ ✓	5

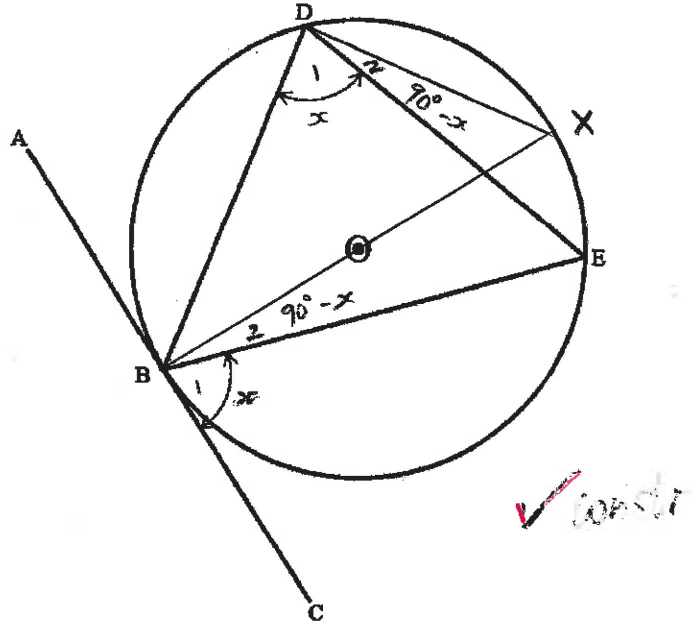
10.3.



	let $\hat{A} = x$	
	$\therefore \hat{F}_1 = x$ ✓SR corr ∠'s =, FD ∥ AC	
	$\therefore E\hat{G}D = x$ ✓S ✓R ∠'s in same ⊙ segm =	
	$\therefore \hat{B} = x$ ✓SR corr ∠'s =, GD ∥ BC	
	$\therefore \hat{A} = \hat{B}$ both $= x$	
	$\therefore$ AECB is a cyclic quad conv ∠'s in same ⊙ segm = ✓R	5

QUESTION 11

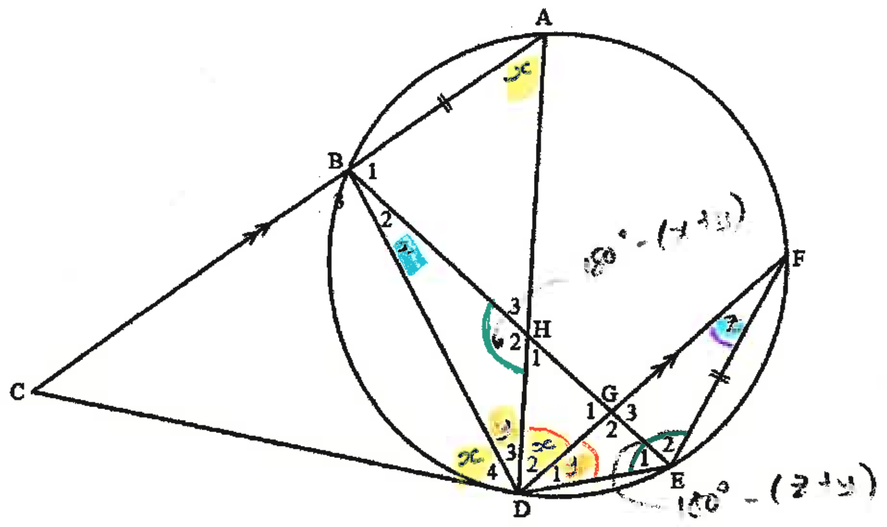
11.1.



✓ constr

	let $\hat{B}_1 = x$	
	$\therefore \hat{B}_2 = 90° - x$ ✓ ✓ tan ⊥ rad	
	$\therefore \hat{D}_2 = 90° - x$ ✓SR ∠'s in same ⊙ segm =	
	$\therefore \hat{D}_1 = x$ ✓SR ∠ in semi ⊙ = 90°	
	$\therefore \hat{B}_1 = \hat{D}_1$ both $= x$	
	$\therefore E\hat{B}C = B\hat{D}E$	5
	$\hat{B}_1 + \hat{B}_2 = 90°$ ✓S ✓R tan ⊥ rad	
	$\hat{D}_1 + \hat{D}_2 = 90°$ ✓SR ∠ in semi ⊙ = 90°	
	but $\hat{B}_2 = \hat{D}_2$ ✓SR ∠'s in same ⊙ segm =	
	$\therefore \hat{B}_1 = \hat{D}_1$	

11.2.



11.2.1.	Let $\hat{D}_2 = x$	
	$\therefore \hat{A} = x$ ✓SF alt ∠'s =, CA ∥ DF	
	$\therefore \hat{D}_4 = x$ ✓✓F ∠ tan chord	
	$\therefore \hat{D}_2 = \hat{D}_4$ both $= x$	3
11.2.2.	Let $\hat{D}_1 = y$	
	$\therefore \hat{D}_3 = y$ ✓✓ = chords = ∠'s @ circum	
	let $\hat{B} = z$	
	$\therefore \hat{D} = z$ ✓ ∠'s in same ⊙ segm =	
	$\hat{H}_2 = 180° - (z+y)$ ✓SF	
	sum ∠'s in Δ = 180°	

	Similarly, $\hat{E}_1 + \hat{E}_2 = 180° - (z+y)$ ✓S	
	$\therefore \hat{H}_2 = \hat{E}_1 + \hat{E}_2$ both $= 180° - (z+y)$	5